

A9: Pulley System Design

- Center Distance 4-6 in
- Pulley Diam = 3 in
- Rotational Speed = 1,000 - 2,000 rpm
- Belt Tension (Static) = 4-6 lbf
- Mat'l = Al, ABS, or Steel Pulleys
- Safety Factor = minimum 3
- belt = polyurethane rubber
- $LS_y = 3.25 \text{ MPa (471 psi) at } 20\% \text{ strain}$

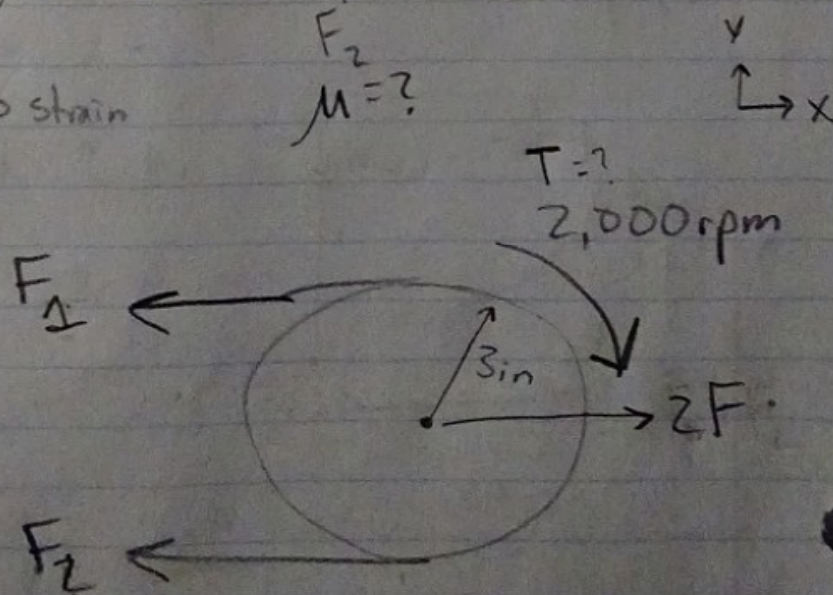
Belt Selection

① Calculate forces on both tight and loose side of belt with no slippage.

Calculate σ using normal stress eq. & yield strength.
Calculate belt's outer circumference then select a suitable belt

known
 pulley $\phi = 3 \text{ in}$
 tension = 6 lbf
 rpm = 2,000 rpm
 $S_y = 471 \text{ psi @ } 20\% \text{ strain}$
 $SF = 3$
 no slippage
 $\theta = \pi$

unknown
 belt $\phi = \phi_B$
 F_1
 F_2
 $\mu = ?$



$$\frac{S_y}{SF} = \frac{F}{A}$$

Statically, $F_1 = F_2 = \text{belt tension} = 6 \text{ lbf}$

$$A_{\text{belt section}} = \frac{F(SF)}{S_y} = \frac{6 \text{ lbf} (3)}{471 \text{ psi}}$$

$$A_{\text{belt}} = 0.0382 \text{ in}^2$$

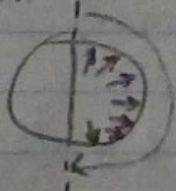
$$A = \pi \frac{d^2}{4} \quad d = \phi_B = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.0382)}{\pi}}$$

$$\phi_B = 0.221 \text{ in} \approx 1/4 \text{ in}$$

Dynamically, $F_{\text{Tight}} > F_{\text{loose}}$

$$P = T\omega = r\omega(F_{\text{Tight}} - F_{\text{loose}})$$

$\uparrow$ unknown $\uparrow$ known $\uparrow$ known $\uparrow$ unknown

$$\frac{F_1}{F_2} = e^{\mu\Theta}$$


$\Theta = 180^\circ = \pi$

$$F_1 + F_2 = 2F = 2(6 \text{ lbf}) = 12 \text{ lbf}$$

www.engineersedge.com/coefficients-of-friction.htm

"rubber (60 A belt) — stainless steel 316 $\rightarrow \mu_s = 0.64$

Assumption: Mat'l pulley = "Steel"
 $\mu_s \approx 0.64$

$$F_1 + F_2 = 2F$$

$$\frac{F_1}{F_2} = e^{\mu\theta}$$

$$F_1 = F_2 e^{\mu\theta}$$

$$F_2 (e^{\mu\theta} + 1) = 2F$$

$$F_2 = \frac{2F}{e^{\mu\theta} + 1} \quad F_1 = 2F - F_2$$

$$F_2 = \frac{2(6 \text{ lbf})}{e^{(0.64)(\pi)} + 1} = 1.417 \text{ lbf}$$

$$F_1 = 2(6 \text{ lbf}) - (1.417 \text{ lbf}) = 10.58 \text{ lbf}$$

$$F_{\text{tight}} = 10.58 \text{ lbf (max)}$$

$$F_{\text{loose}} = 1.417 \text{ lbf (min)}$$

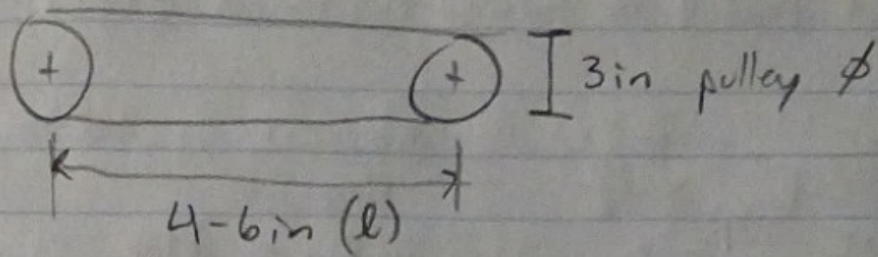
$$\frac{S_y}{SF} = \frac{F_{\text{tight}}}{A} \rightarrow A = \frac{F_{\text{tight}}(SF)}{S_y} = \pi \frac{d^2}{4}$$

$$\phi_B = \sqrt{\frac{4 F_{\text{tight}}(SF)}{\pi S_y}} = \sqrt{\frac{4(10.58 \text{ lbf})(3)}{\pi (471 \text{ psi})}}$$

$$\phi_B = 0.293 \text{ in}$$

$$\boxed{5/16''} = 0.3125 \text{ in}$$

Belt Circumference



using max $\rightarrow 6$ in

$$\text{Belt circumference} = 2l + \pi \phi_{\text{pulley}}$$

$$C_B = 2(6 \text{ in}) + \pi(3 \text{ in}) = 21.4 \text{ in}$$

$$\boxed{\text{Circumference of Belt} = 21.4 \text{ in}}$$

Belt selection McMaster-Carr

$$\phi_B = 0.243 \text{ in} \approx 5/16''$$

$$\phi_B = 0.243 \text{ in} \left(\frac{1 \text{ cm}}{2.54 \text{ in}} \right) = 0.1154 \text{ cm} = 1.154 \text{ mm} \approx 2 \text{ mm}$$

$$C_B = 21.4 \text{ in}$$

$$C_B = 21.4 \text{ in} \left(\frac{1 \text{ cm}}{2.54 \text{ in}} \right) \left(\frac{10 \text{ mm}}{1 \text{ cm}} \right) = 84.25 \text{ mm}$$

$$\phi_{\text{pulley}} = 3 \text{ in}$$

$$\phi_{\text{pulley}} = 3 \text{ in} \left(\frac{10 \text{ mm}}{2.54 \text{ in}} \right) = 11.81 \text{ mm}$$

Round Belting, $5/16''$ 59725K733

+ connector 6180K15

Shaft Diameter (minimum)

Shaft mat'l = 1060 hardened steel

known

$$\max T = F_{\text{Tight}} \times \frac{\phi_p}{2}$$

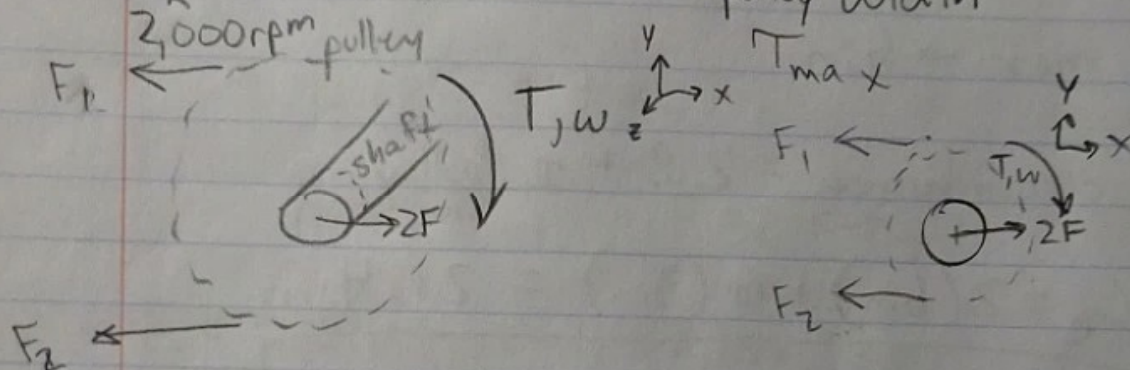
$$F_R = 2F \quad SF=3$$

3000rpm pulley

unknown

ϕ_{shaft}

pulley width



Matweb $\rightarrow$ 1060 Steel, oil quenched 845°C, 480°C temp,...

Shear Modulus = 11600 ksi

tensile $S_y = 98200 \text{ psi} = 98.2 \text{ ksi}$

Research $\rightarrow$ von Mises criterion $\rightarrow \tau_y \approx 0.577 \sigma_y$

$$\tau_y = 0.577 (98.2 \text{ ksi}) = 56.7 \text{ ksi}$$

$$\tau_y = 56,700 \text{ psi}$$

$$\frac{\tau_y}{SF} = \frac{V}{A} = \frac{2F}{A_{\text{surface}}}$$

$A_s = \text{pulley width} \times \text{circumference shaft}$

Diameter from Torque

$$\text{Torque} = F_{\text{TIGHT}} \frac{\phi p}{2} = 10.58 \text{ lb}_f \left(\frac{3 \text{ in}}{2} \right)$$

$$\text{Torque} = 15.87 \text{ in-lb}_f$$

$$\tau = \frac{T r}{J} = \frac{T \frac{\phi}{2}}{\frac{\pi \phi^4}{32}} = \frac{16 T}{\pi \phi^3}$$

$$\phi^3 = \frac{16 T}{\pi \tau}$$

$$\phi = \sqrt[3]{\frac{16 T}{\pi \tau}}$$

$$\phi = \sqrt[3]{\frac{16 (10.58 \text{ in-lb}_f)}{\pi (56,700 \text{ psi})}} = 0.0983 \text{ in}$$

$$\phi_{\text{shaft}} \approx 0.0983 \text{ in}$$

options likely $\frac{1}{8}'' = 0.125 \text{ in}$

Journal Bearing Design

mat'l = sintered metal or non-metallic

Table 4 (²⁴⁴²~~pg 2440~~) Machinery's Handbook

Determine P , V , PV , select suitable material

Known

2000 rpm

$T = 15.87 \text{ in-lbf}$

$\phi_{\text{shaft}} = 1/8" = 0.125$

Unknown

P

Bearing depth (l_B)

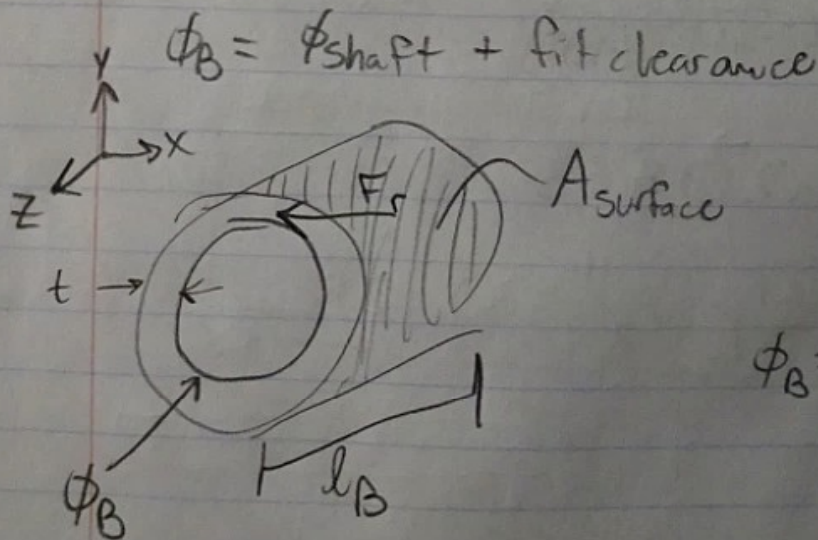
V

ϕ_{bearing}

PV

F_r

Assumption: RC5 fit and $l_B = 1 \text{ in}$
hole $+0.7 \text{ thou}$



$$\phi_B = 1/8" + 0.0007 \text{ in}$$

$$\phi_B = 0.1257 \text{ in}$$

$$P = \frac{F}{A} = \frac{F_r}{A} = \frac{2T/\phi_B}{l_B \times \pi \phi_B} = \frac{2T}{l_B \pi \phi_B^2}$$

$$P = \frac{2(15.87 \text{ in-lbf})}{(1 \text{ in}) \pi (0.1257 \text{ in})^2} = 639 \text{ psi}$$

$$V = (\text{rpm}) \left(\frac{\pi \phi_B}{1 \text{ rev}} \right) \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) \phi_B - \text{in}$$

$$V = (2,000) \left(\pi (0.1257 \text{ in}) \right) \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) = 65.8 \text{ ft/min}$$

$$P = 634 \text{ psi}$$

$$V = 65.8 \text{ ft/min}$$

$$PV = 42,046.2 \text{ psi-ft/min}$$

Options

Porous Bronze

Porous iron

Porous metals

Porous Bronze

$$P \leq 4500 \text{ psi}$$

✓

$$V \leq 1500 \text{ ft/min}$$

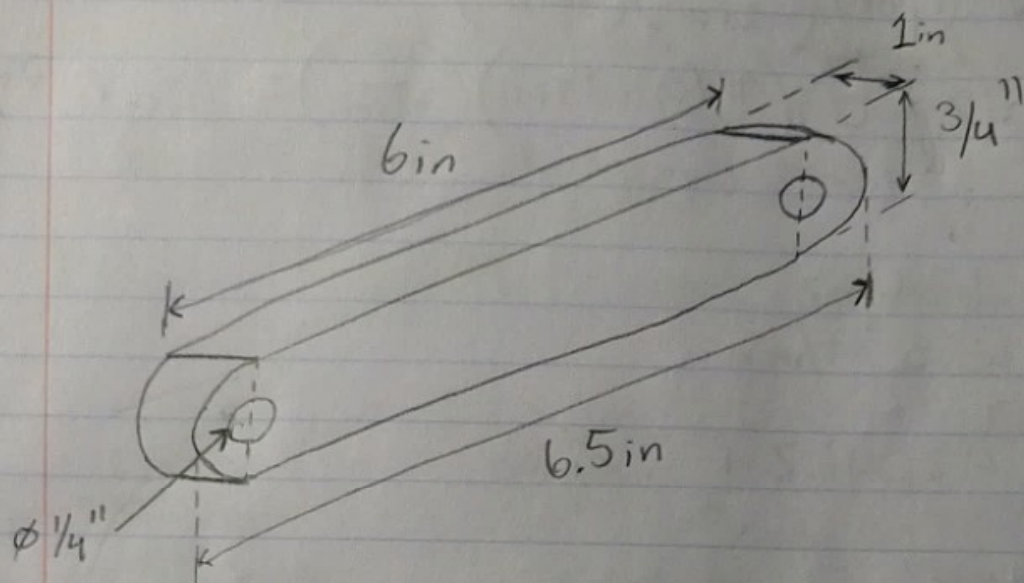
✓

$$PV \leq 50,000 \text{ psi-ft/min}$$

✓

Assumption: Porous Bronze uses a sintered process

Design a Frame



journal bearings + shaft $\sim 1/4''$ diameter hole