A2

- A500 structural steel

P = 15kN   a = 0.4m   b = 0.3m
A = pin joint   B = roller joint

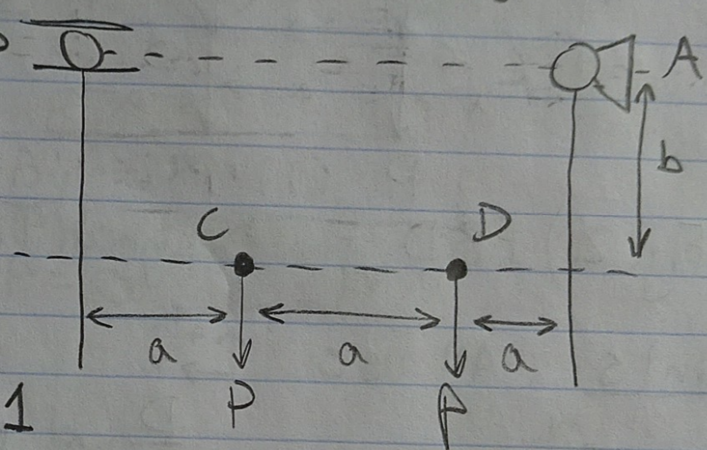
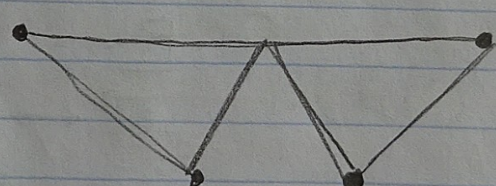


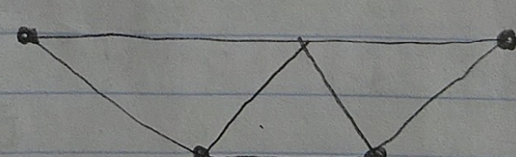
Fig. 1

Designs

1.

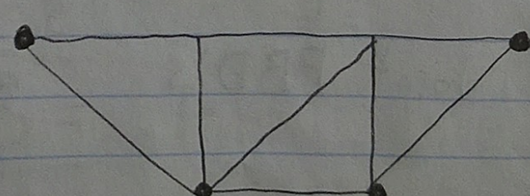


2.



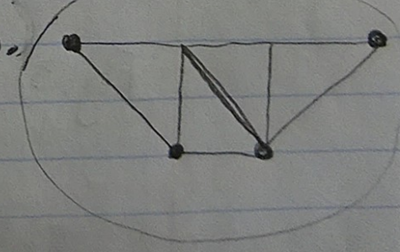
most support force closest to pin A is spread better

3. a.



or

b.



☆FBD = Free Body Diagram

Chosen Truss Geometry (3b)

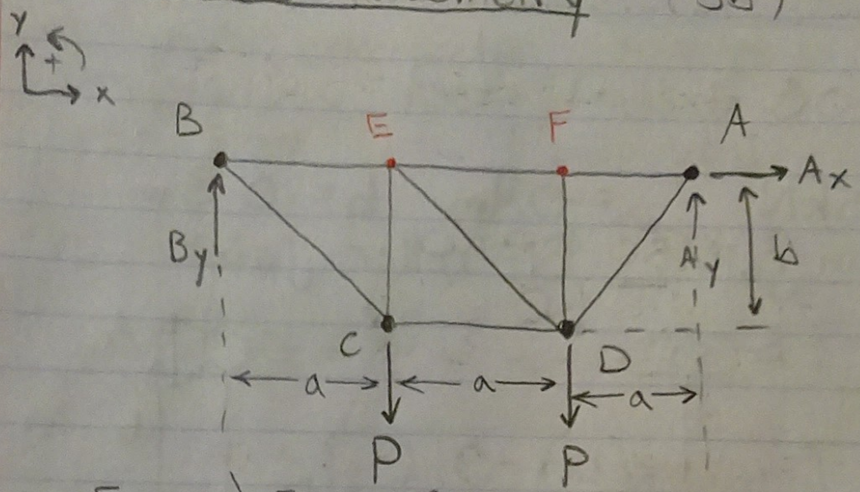


Fig. 2) FBD for Designed Truss Geometry

Joints All use

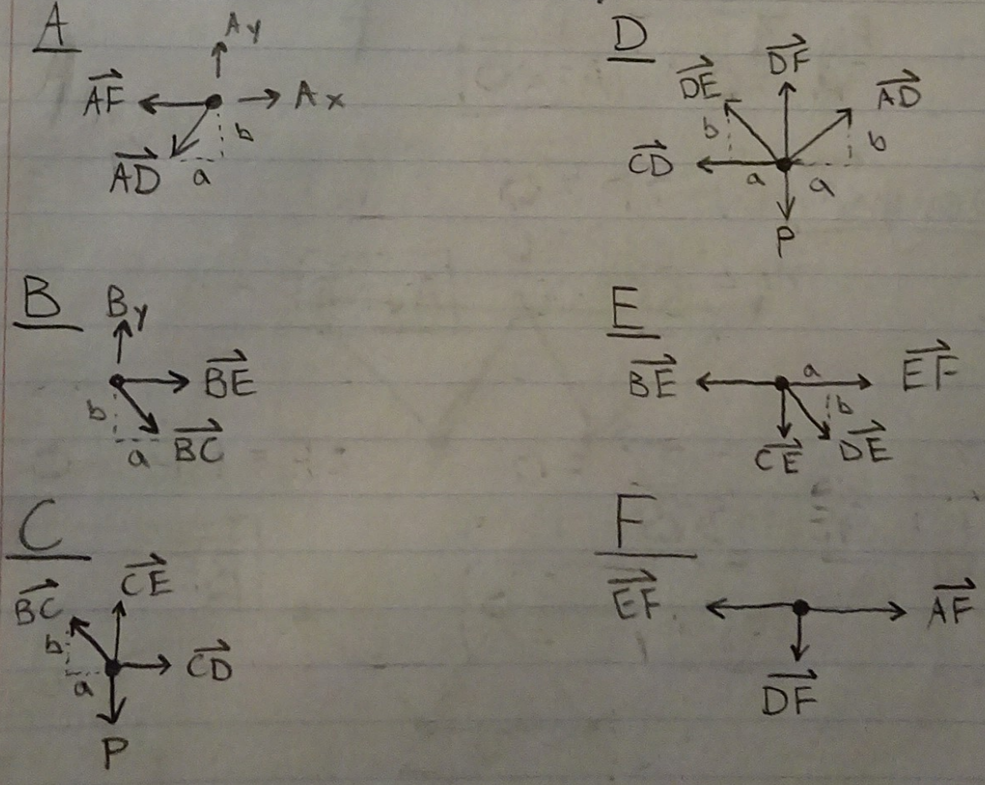


Fig. 3) All Joint FBD's

Solving Internal Forces Symbolically

for all diagonal members $c = \sqrt{a^2+b^2}$

External Forces

$\Sigma F_x = \boxed{A_x = 0}$

$\Sigma M_A = P(a) + P(2a) - B_y(3a) = 0$

$P(3a) = B_y(3a)$

$\boxed{B_y = P}$

$\Sigma F_y = A_y + B_y - 2P = 0$

$A_y + (P) - 2P = 0$

$A_y - P = 0 \quad \boxed{A_y = P}$

B

$\Sigma F_y = B_y - \frac{b}{c}\overrightarrow{BC} = 0$

$B_y = \frac{b}{c}\overrightarrow{BC}$

$\overrightarrow{BC} = \frac{c}{b} B_y \quad \boxed{\overrightarrow{BC} = \frac{c}{b}P}$

$\Sigma F_x = \overrightarrow{BE} + \frac{a}{c}\overrightarrow{BC} = 0$

$\overrightarrow{BE} = -\frac{a}{c}\overrightarrow{BC} = -\frac{a}{c}\left(\frac{c}{b}P\right) \quad \boxed{\overrightarrow{BE} = -\frac{a}{b}P}$

C

$\Sigma F_y = \overrightarrow{CE} + \frac{b}{c}\overrightarrow{BC} - P = 0$

$\overrightarrow{CE} + \frac{b}{c}\left(\frac{c}{b}P\right) - P = 0$

$\overrightarrow{CE} + P - P = 0 \quad \boxed{\overrightarrow{CE} = 0}$

$\Sigma F_x = \overrightarrow{CD} - \frac{a}{c}\overrightarrow{BC} = 0$

$\overrightarrow{CD} - \frac{a}{c}\left(\frac{c}{b}\right)P = 0$

$\boxed{\overrightarrow{CD} = \frac{a}{b}P}$

E

$$\Sigma F_y = -\overrightarrow{CE} - \tfrac{b}{c}\overrightarrow{DE} = 0$$
$$-(0) - \tfrac{b}{c}\overrightarrow{DE} = 0 \qquad \boxed{\overrightarrow{DE} = 0}$$

$$\Sigma F_x = \overrightarrow{EF} + \tfrac{a}{c}\overrightarrow{DE} - \overrightarrow{BE} = 0$$
$$\overrightarrow{EF} + \tfrac{a}{c}(0) - \left(-\tfrac{a}{b}P\right) = 0$$
$$\overrightarrow{EF} + \tfrac{a}{b}P = 0 \qquad \boxed{\overrightarrow{EF} = -\tfrac{a}{b}P}$$

F

$$\Sigma F_y = \boxed{\overrightarrow{DF} = 0}$$

$$\Sigma F_x = \overrightarrow{AF} - \overrightarrow{EF} = 0$$
$$\overrightarrow{AF} - \left(-\tfrac{a}{b}P\right) = 0 \qquad \boxed{\overrightarrow{AF} = -\tfrac{a}{b}P}$$

A

$$\Sigma F_y = A_y - \tfrac{b}{c}\overrightarrow{AD} = 0$$
$$P - \tfrac{b}{c}\overrightarrow{AD} = 0$$
$$P = \tfrac{b}{c}\overrightarrow{AD} \qquad \boxed{\overrightarrow{AD} = \tfrac{c}{b}P}$$

$$\Sigma F_x = A_x - \overrightarrow{AF} - \tfrac{a}{c}\overrightarrow{AD} = 0$$
$$0 - \left(-\tfrac{a}{b}P\right) - \tfrac{a}{c}\overrightarrow{AD} = 0$$
$$-\tfrac{a}{c}\overrightarrow{AD} = -\tfrac{a}{b}P \qquad \overrightarrow{AD} = \tfrac{c}{b}P \checkmark$$

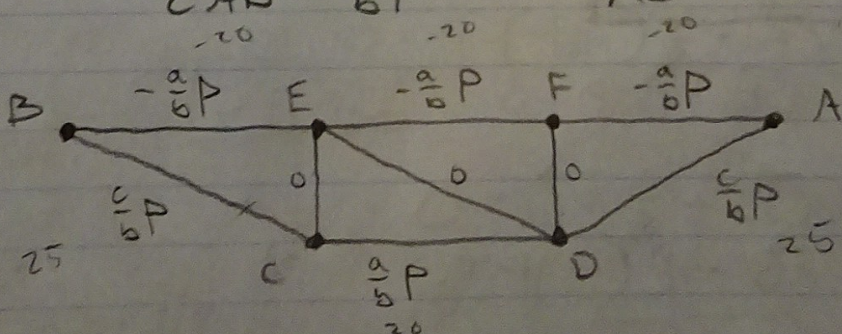


Fig. 4) Forces shown on truss

Internal Forces

$\vec{BC} = \frac{c}{b}P$

$\vec{BE} = -\frac{a}{b}P$

$\vec{CE} = 0$

$\vec{CD} = \frac{a}{b}P$

$\vec{DE} = 0$

$\vec{EF} = -\frac{a}{b}P$

$\vec{DF} = 0$

$\vec{AF} = -\frac{a}{b}P$

$\vec{AD} = \frac{c}{b}P$

Numerically Solving

$a = 0.4m$ $\quad P = 15kN$

$b = 0.3m$

$c = \sqrt{a^2+b^2} = \sqrt{0.4^2+0.3^2} = 0.5m$

$\vec{BC} = 25kN\ (T)$

$\vec{BE} = 20kN\ (C)$

$\vec{CE} = 0\ kN$

$\vec{CD} = 20kN\ (T)$

$\vec{DE} = 0\ kN$

$\vec{EF} = 20kN\ (C)$

$\vec{DF} = 0\ kN$

$\vec{AF} = 20\ kN\ (C)$

$\vec{AD} = 25\ kN\ (C)$

3 Members are = 0, the design can be optimized/reduced

Truss Geometry (2)

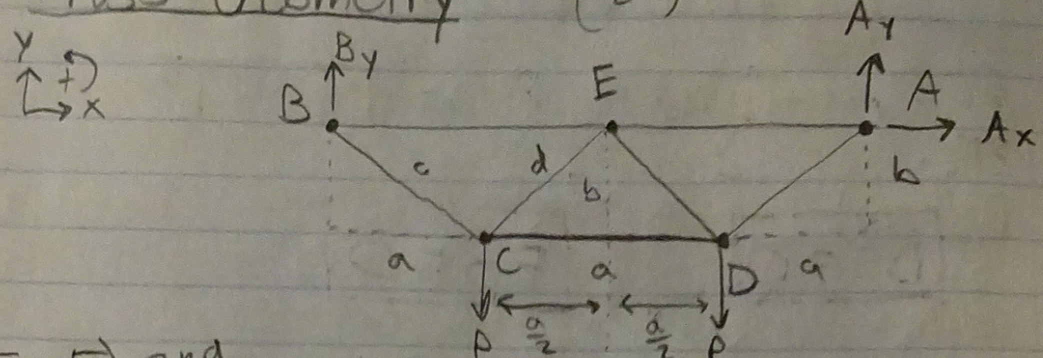


Fig 5) 2nd Attempt FBD for Designed Truss Geometry

Joints All use y $+$ x

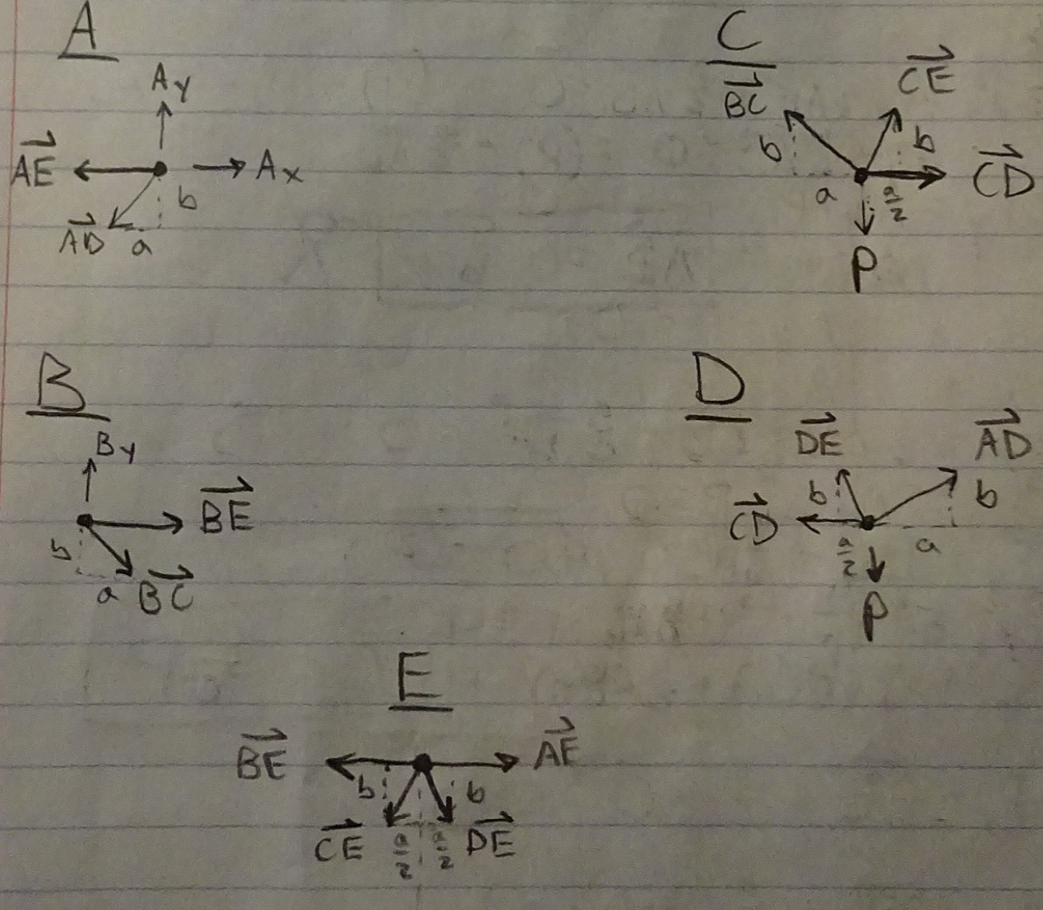


Fig 6) 2nd Attempt All Joint FBD's

Solving Internal Forces Symbolically

$$c = \sqrt{a^2+b^2} \qquad d = \sqrt{\left(\frac{a}{2}\right)^2 + b^2}$$

External Forces

$$\Sigma M_A = P(a) + P(2a) - B_y(3a) = 0$$

$$P(3a) = B_y(3a)$$

$$\boxed{B_y = P}$$

$$\Sigma F_x = \boxed{A_x = 0}$$

$$\Sigma F_y = A_y + B_y - 2P = 0$$

$$A_y + P - 2P = 0$$

$$A_y - P = 0$$

$$\boxed{A_y = P}$$

B

$$\Sigma F_y = B_y - \frac{b}{c}\overrightarrow{BC} = 0$$

$$(P) = \frac{b}{c}\overrightarrow{BC} \qquad \boxed{\overrightarrow{BC} = \frac{c}{b}P}$$

$$\Sigma F_x = \overrightarrow{BE} + \frac{a}{c}\overrightarrow{BC} = 0$$

$$\overrightarrow{BE} + \frac{a}{c}\left(\frac{c}{b}P\right) = 0$$

$$\boxed{\overrightarrow{BE} = -\frac{a}{b}P}$$

$\underline{C}$

$$\Sigma F_y = \frac{b}{c}\overrightarrow{BC} + \frac{b}{d}\overrightarrow{CE} - P = 0$$

$$\frac{b}{c}\left(\frac{c}{b}P\right) + \frac{b}{d}\overrightarrow{CE} - P = 0$$

$$P + \frac{b}{d}\overrightarrow{CE} - P = 0 \qquad \boxed{\overrightarrow{CE} = 0}$$

$$\Sigma F_x = \overrightarrow{CD} + \frac{a}{2d}\overrightarrow{CE} - \frac{a}{c}\overrightarrow{BC} = 0$$

$$\overrightarrow{CD} + \frac{a}{2d}(0) - \frac{a}{c}\left(\frac{c}{b}P\right) = 0$$

$$\overrightarrow{CD} - \frac{a}{b}P = 0 \qquad \boxed{\overrightarrow{CD} = \frac{a}{b}P}$$

Symmetric Design

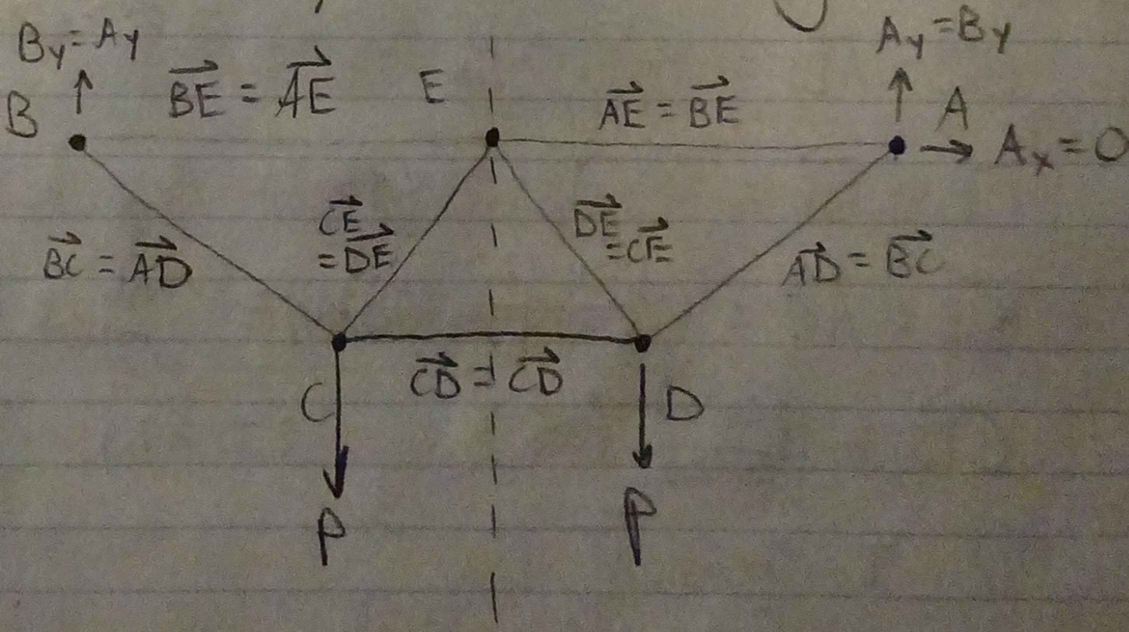


Fig. 7) FBD showing symmetric forces

Internal Forces

$\overrightarrow{BC} = \overrightarrow{AD} = \frac{c}{b} P$

$\overrightarrow{CE} = \overrightarrow{DE} = 0 P$ 7 members

$\overrightarrow{BE} = \overrightarrow{AE} = -\frac{a}{b} P$

$\overrightarrow{CD} = \frac{a}{b} P$

Numerically Solving

$a = 0.4m$ $P = 15kN = 15{,}000N$

$b = 0.3m$

$c = \sqrt{a^2+b^2} = \sqrt{0.4^2+0.3^2} = 0.5m$

$d = \sqrt{(\frac{a}{2})^2+b^2} = \sqrt{0.2^2+0.3^2} = 0.361m$

$\overrightarrow{BC} = \overrightarrow{AD} = \frac{(0.5m)}{(0.3m)}(15kN) = 25 \ kN \ (T)$

$\overrightarrow{CE} = \overrightarrow{DE} = 0 = 0 kN$

$\overrightarrow{BE} = \overrightarrow{AE} = -\frac{(0.4m)}{(0.3m)}(15kN) = 20 \ kN \ (C)$

$\overrightarrow{CD} = \frac{(0.4)}{(0.3)}(15) = 20kN \ (T)$

T = Tension
C = Compression

2nd Attempt does not change the max forces
reduces 2 members and 1 joint

2nd is better

Calculating Cross-Section Area

$SF = 3.5$ (given) Material - A500 (not in SW)

Substituting with ASTM A36 (lower S_y)

(SolidWorks)
ASTM A36 Steel
- $S_y = 250\ N/mm^2$ or $250\ MPa$

$\sigma = \frac{S_y}{SF}$ $\sigma = \frac{F}{A}$ $F = 25kN$ (from max force in truss)

$\frac{S_y}{SF} = \frac{F}{A}$ $A = ?$

$A_{min} = F\left(\frac{SF}{S_y}\right)$

$A_{min}\ (mm^2) = F(N)\left(\frac{SF}{S_y(MPa)}\right)$

$A_{min}\ (mm^2) = (25{,}000\ N)\left(\frac{3.5}{250MPa}\right)$

$\boxed{A_{min} = 350\ mm^2}$

Approximate Weight of the Truss

cross-sectional Area $= 350\,mm^2$

(from SolidWorks) mass density $= 7850\,kg/m^3$

Truss member lengths

CD $1 \times a$ or $1\times$ $0.4m$

BC+AD $2 \times c$ or $2\times$ $0.5m$

CE+DE $2 \times d$ or $2\times$ $0.361m$

BE+AE $2 \times \frac{3a}{2}$ or $2\times$ $0.6m$

Total Length $= 0.4 + 2(0.5) + 2(0.361) + 2(0.6)$

$L_{tot} = 3.322\,m$

$A = 350\,mm^2 \left(\frac{m}{1{,}000mm}\right)^2 = 0.00035\,m^2$

$V = 3.322\,m \times 0.00035\,m^2$

$V = 0.0011627\,m^3$

$\rho = \frac{m}{V} \rightarrow m = \rho V$

$m = \left(7850\,\frac{kg}{m^3}\right)\left(0.0011627\,m^3\right)$

$m = 9.127\,kg$ $W = mg = (9.127kg)(9.81\,m/s^2)$

$\boxed{W = 89.5\,N}$

Determining Pin Cross-Sections

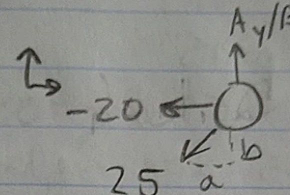
Given: shear yield strength $SS_y = 170$ ksi

☆ single-shear $\rho = 0.278\ lb/in^3$

$SF = 4$

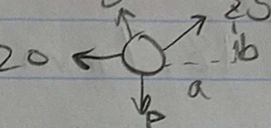
$F_\tau = \sqrt{F_x^2 + F_y^2}$

Pin A+B



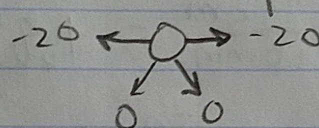
$F_\tau = 25\,kN$

Pin C+D



$\Sigma F_\tau = 25\,kN = 40\,kN$

Pin E



$\Sigma F_x = 20\,kN$

Max Shear Force = 25 kN

$25\,kN = 25{,}000\,N \left(\frac{0.2248\,lb}{1\,N}\right) = 5620\,lb$

$170\,ksi = 170{,}000\,psi$

$\frac{SS_y}{SF} = \tau \qquad \tau = \frac{F}{A}$ (single shear)

$\frac{SS_y}{SF} = \frac{F}{A} \Rightarrow A_{min} = F\left(\frac{SF}{SS_y}\right)$

$d = 2\sqrt{\frac{A}{\pi}}$

~~$d = 0.41\,in$~~

$A_{min} = (5620\,lb)\left(\frac{4}{170{,}000\,psi}\right) = \boxed{A_{min} = 0.132\,in^2}$

$A_{min} = 0.132\,in^2 \left(\frac{2.54\,cm}{1\,in}\right)^2 \left(\frac{10\,m}{1\,cm}\right)^2 = 85.2\,mm^2$

$A = \pi\left(\frac{d}{2}\right)^2 \qquad d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{136.8\,mm^2}{\pi}}$

$d = 2r = \boxed{d = 10.4\,mm}$

Combined Weight of pins

$\rho = 0.278\ lb/in^3$

$A_{min} = 0.132 in^2$

length of the pins

Assume members are square

$A = 350 mm^2$

$s = \sqrt{A} = 18.7 mm \longrightarrow$ center $\frac{18.7mm}{2} = 9.35mm$

$18.7 mm \times \frac{1 cm}{10 mm} = 1.87 cm$

$1.87 cm \times \frac{1 in}{2.54 cm} = 0.736 in$

☆ According to directions for CAD modeling intersection of pin joint must maintain the cross-sectional area of each element

∴ pin length = 1 side of square member

length = 0.736 in

$V_{pin} = 0.736 in \times 0.132 in^2 = 0.0972 in^3$

5 pins total $V \times 5 = 0.486 in^3$

$\rho = \frac{m}{V}$ $m = \rho V = \left(0.278 \frac{lb}{in^3}\right)\left(0.486 in^3\right)$

$\boxed{m/W = 0.135 lb}$

did not state lb_m or lb_f so I'll assume lb_f

since $1 lb_m = 1 lb$

Solid Works

- Accidentally made pin holes too small
  - miscalculated in^3 to mm^3
- mass properties gives mass = 8778.20 g

$9{,}007.11\,g = 9.00711\,kg \times 9.81\,m/s^2$

$W = 88.4\,N$ close to approximation (both are approximations)

$W_{truss} + W_{pins} = W_{total}$

$W_{pins} = 0.135\,lb\left(\frac{4.45\,N}{1\,lb}\right) = 0.60\,N$

$(89.5\,N) + (0.60\,N) = 90.1\,N$

Total Calculated Weight = 90.1 N
Estimated Weight in SW = 88.4 N

$\%\,Difference = \frac{SW\,estimate - calculated}{calculated} \times 100\%$

$\%\,Dif = -1.89\%$